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XOR nonlocal games, their sequential monopartite counterparts and sources of quantum advantage

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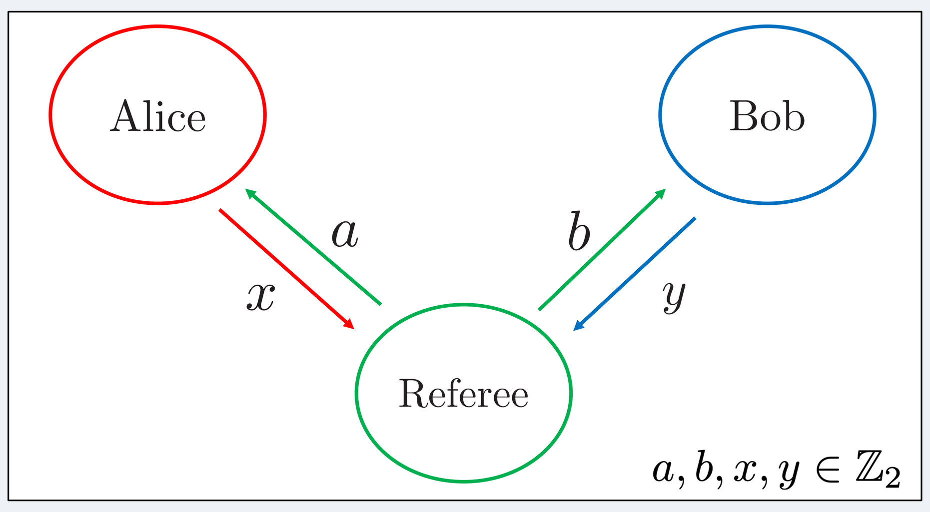
Introduction

In this work we focus on two classes of games: XOR nonlocal games and XOR* sequential games with monopartite resources. XOR games have been widely studied in the literature of nonlocal games, and we introduce XOR* games as their natural counterpart within the setup of sequential games with monopartite resources. Examples of XOR* games are 2 to 1 quantum random access codes (QRAC), 2 to 1 parity oblivious multiplexing and the CHSH* game. We relate these two classes of games via an explicit theorem that connects their optimal strategies, and so their classical (Bell) and quantum (Tsirelson) bounds. The two classes of games that we consider include most of the already studied games in the literature, and the connection that we establish provides a key to understand the relationship between the nonclassical resources responsible for the quantum-over-classical computational advantage. More precisely, the mapping between the two classes of games turns established proofs of nonlocality in XOR games into proofs of preparation contextuality in XOR* games.

Paradigmatic example: CHSH and CHSH* games

CHSH game



Goal: $x + y = a \cdot b$.

Setting	Optimal performance
Classical	0.75
Quantum	$\cos^2(\frac{\pi}{8})$
Boxworld	1

CHSH game*

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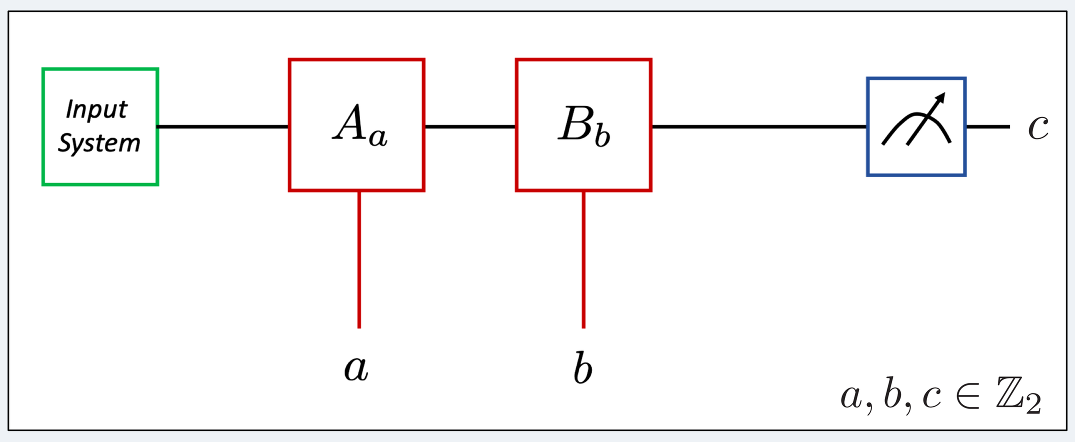
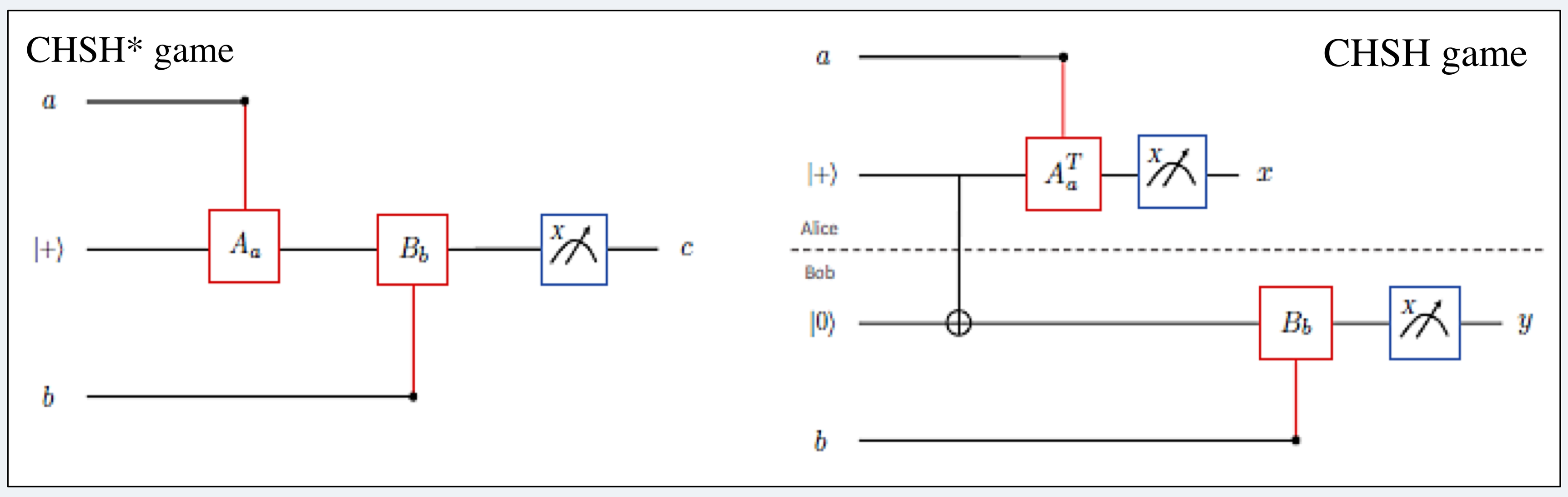


Goal: $c = a \cdot b$.

Setting (d=2)	Optimal performance
Classical reversible	0.75
Quantum reversible	$\cos^2(\frac{\pi}{8})$
Irreversible	1

Mapping

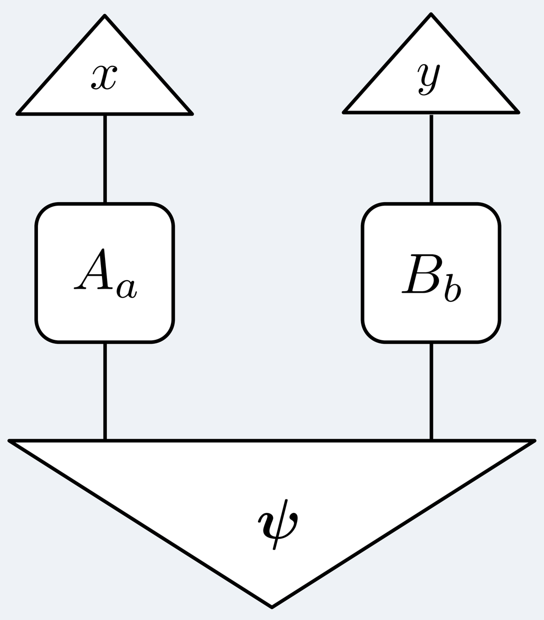
Theorem. Any strategy in the CHSH* game can be mapped to a strategy in the CHSH game with the same success probability.



XOR and XOR* games

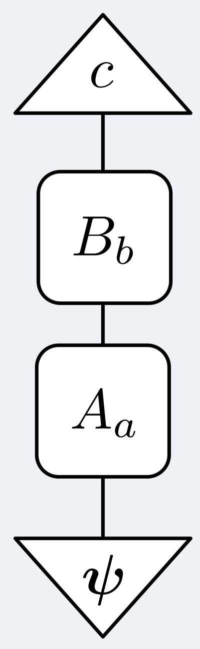
XOR games

- Setup: two inputs $a \in \mathbb{Z}_d^n$ and $b \in \mathbb{Z}_d^m$ sampled from $p(a, b)$ for two players, Alice and Bob, respectively. They can agree on a strategy to output bits $x, y \in \mathbb{Z}_2$. They cannot communicate after the game begins.
- Restrictions: No-signaling,
$$p(x|a, b) = p(x|a); \quad p(y|a, b) = p(y|b).$$
- Winning condition: $f(a, b) = x \oplus y$.



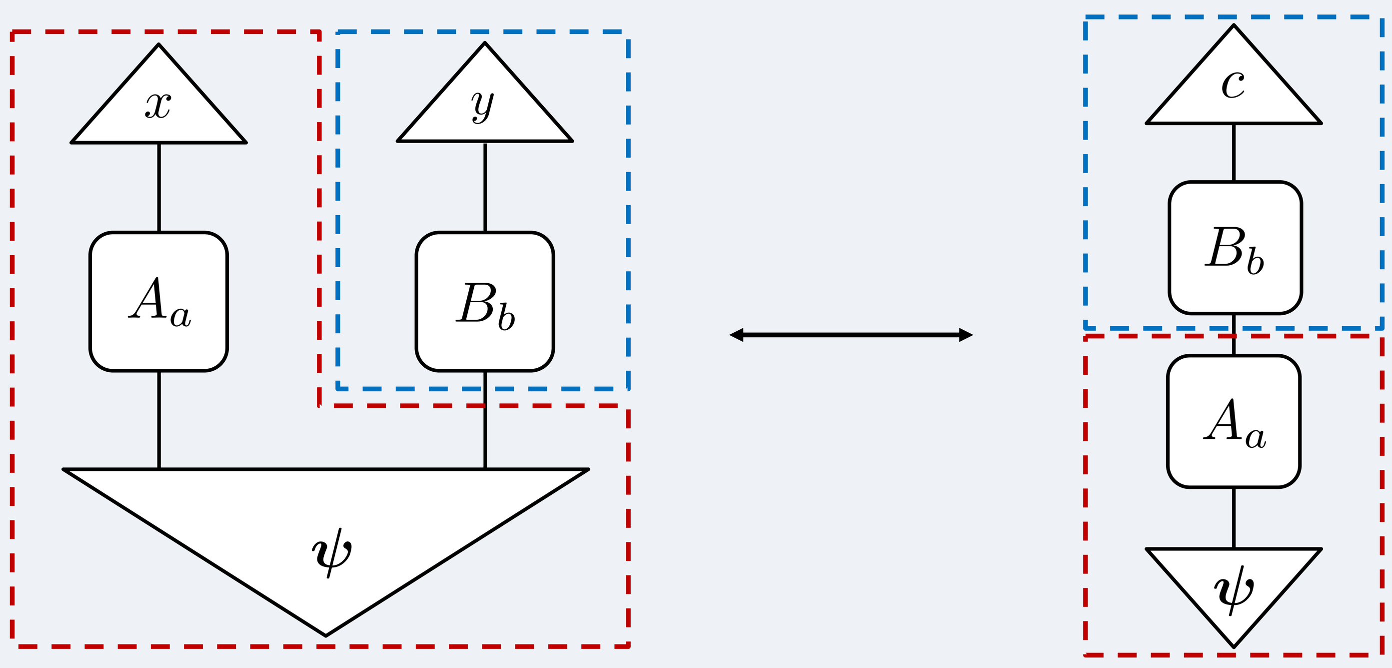
XOR games*

- Setup: two inputs $a \in \mathbb{Z}_d^n$ and $b \in \mathbb{Z}_d^m$ sampled from $p(a, b)$ for two players, Alice and Bob, respectively. A single resource system is given to Alice who applies an operation A_a , and then to Bob who applies B_b . A measurement at the end has to provide output bit $c \in \mathbb{Z}_2$.
- Restrictions: - No signals backwards in time, $p(x|a, b) = p(x|a)$.
  - "Artificial" constraints, *e.g.* dimensional, reversibility, ...
- Winning condition: $f(a, b) = c$.



Mapping

Theorem. Consider two sets $\mathcal{A}$ and $\mathcal{B}$ where $(\min(|\mathcal{A}|, |\mathcal{B}|) \leq 4)$. For every XOR game with output bits x and y, whose inputs a, b are elements of the sets $\mathcal{A}$ and $\mathcal{B}$, there exists an XOR* game with a two dimensional resource system and involving reversible gates that has the same inputs $a \in \mathcal{A}, b \in \mathcal{B}$ and output $c = x \oplus y$ such that the XOR and XOR* games have the same quantum-over-classical advantage. The converse implication also holds.



Sources of computational advantage

- XOR games are known to be powered by nonlocality.
- Our mapping maps proof of nonlocality in Bell scenarios to proof of preparation contextuality in prepare and measure scenarios.
- Therefore XOR* games cast in prepare and measure scenarios are powered by preparation contextuality.

Conclusions and future directions

- We connect XOR and XOR* games.
- Most studied games in the literature fall into these categories.
- The mapping shows that preparation contextuality powers XOR* games.

Future directions

- Generalize further the classes of games that can be connected.
- Use these simple XOR* games to certify quantumness?

References

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