Graphical Language for Quantum Control

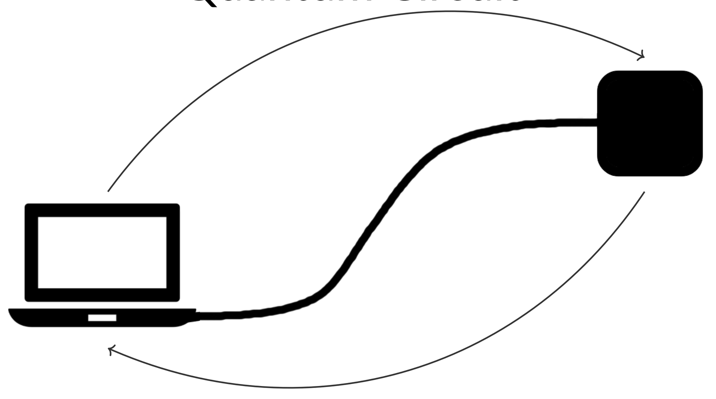
Kostia Chardonnet [1,2] Marc de Visme [2]
Renaud Vilmart [2] Benoît Valiron [2]

[1]Université de Paris, IRIF [2]Université Paris Saclay, LMF

πr^2

The QRAM Model

Quantum Circuit



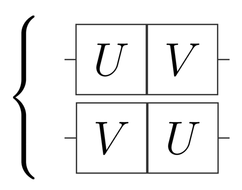
Outcome of measurement

- Classical Computer links to Quantum Co-Processor
- Computer sends Quantum Circuit to Co-Processor
- Co-Processor only handles **tensor of qbits**
- Quantum Circuit semantics akin to **black boxes**
- Control-flow of the program is **classical**

The Quantum Switch

- Two qbits x, y
- Two unitary operations U and V acting on y
- Generate $V \circ U$ if x is $|0\rangle$
- Generate $U \circ V$ if x is $|1\rangle$
- **No duplication** of U and V

$\mathrm{QSwitch}(x, U, V) =$



x can be in superposition!

$(\alpha\,|0\rangle + \beta\,|1\rangle) \otimes |y\rangle \mapsto \alpha\,|0\rangle \otimes (VU\,|y\rangle) + \beta\,|1\rangle \otimes (UV\,|y\rangle).$

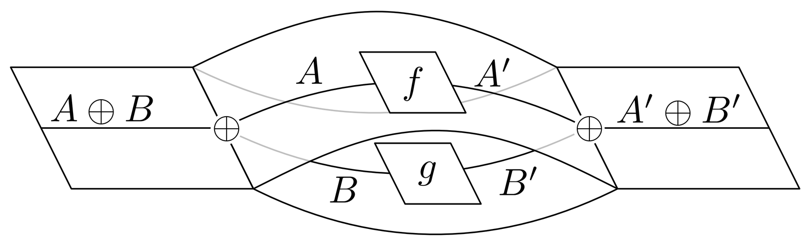
Not capturable by Quantum Circuits, QRAM Model

$\Rightarrow$ But physically realisable!
Need for better formalism

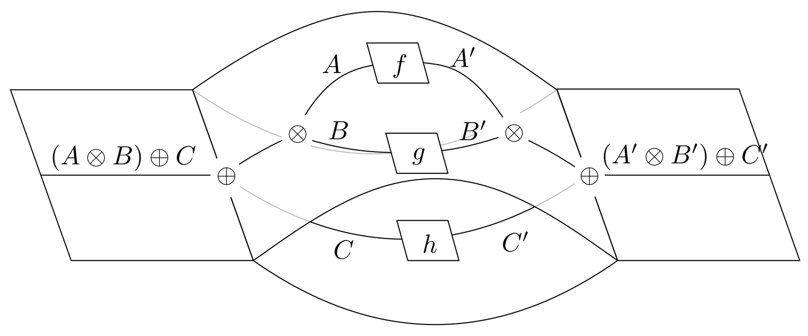
The Multi-World Calculus

- **Clear Semantics**: Denotational and Operational (GoI)
- **Quantum Control**: Possible for every operator
- **Quantum Types**: Not just qbits in superposition

$\Rightarrow$ Symmetric Monoidal Closed Category + Coproduct



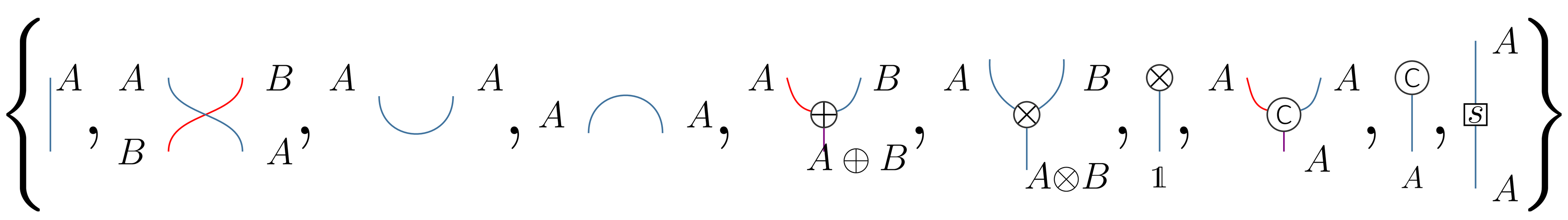
Split over coproduct



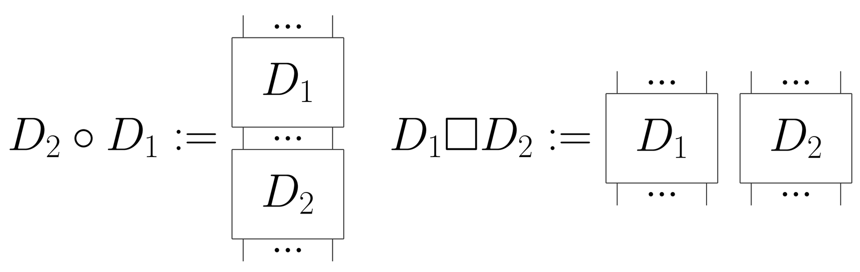
Splits over coproduct and tensor

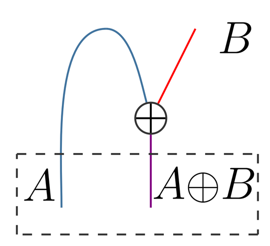
The Graphical Language

Types: $A, B ::= \mathbb{1} \mid A \oplus B \mid A \otimes B$



Composition:



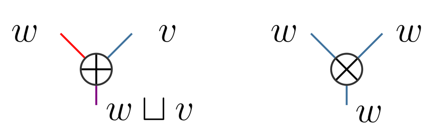


Not a tensor

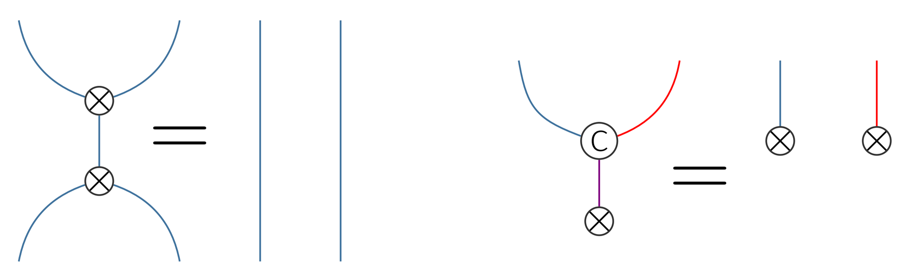
$A \square B :=$ either $(A \otimes B)$ or $(A \oplus B)$

Semantics

- Denotational Semantics $[\![\,.\,]\!] : D \mapsto \mathcal{M}(\mathbb{C})$
- Universality: $\forall f : \mathbb{C}^n \mapsto \mathbb{C}^m, \exists D, [\![D]\!] = f$
- World System:

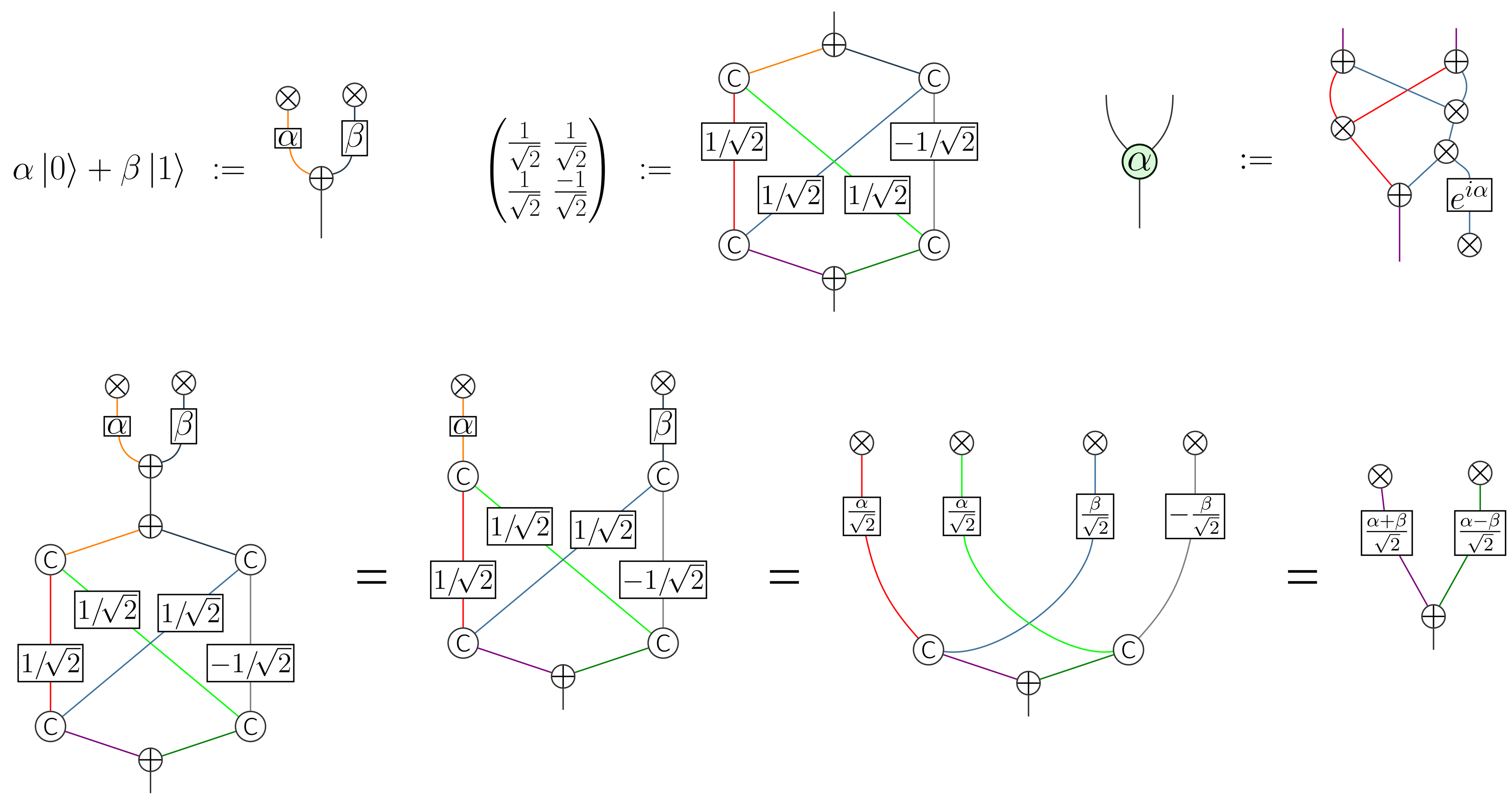


- Equational Theory: ~ 15 axioms



- Soundness & Completeness: $\vdash D = D' \Leftrightarrow [\![D]\!] = [\![D']\!]$

Examples



Back to the Quantum Switch