

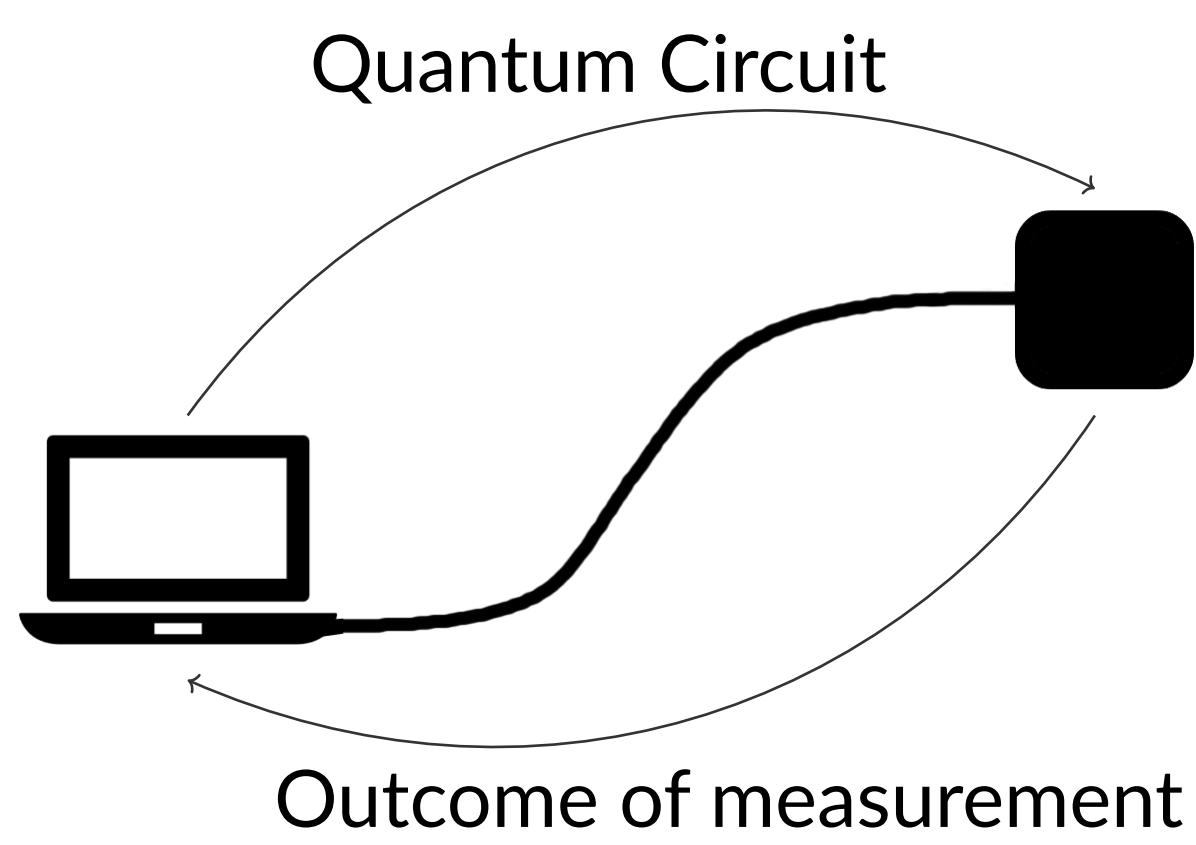
Graphical Language for Quantum Control

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πr^2

The QRAM Model



- Classical Computer links to Quantum Co-Processor
- Computer sends Quantum Circuit to Co-Processor
- Co-Processor only handles **tensor of qubits**
- Quantum Circuit semantics akin to **black boxes**
- Control-flow of the program is **classical**

The Quantum Switch

- Two qubits x, y
- Two unitary operations U and V acting on y
- Generate $V \circ U$ if x is $|0\rangle$
- Generate $U \circ V$ if x is $|1\rangle$
- **No duplication** of U and V

$$\text{QSwitch}(x, U, V) = \begin{cases} \begin{array}{|c|c|} \hline U & V \\ \hline V & U \\ \hline \end{array} & \text{if } x = |0\rangle \\ \begin{array}{|c|c|} \hline V & U \\ \hline U & V \\ \hline \end{array} & \text{if } x = |1\rangle \end{cases}$$

x can be in superposition!

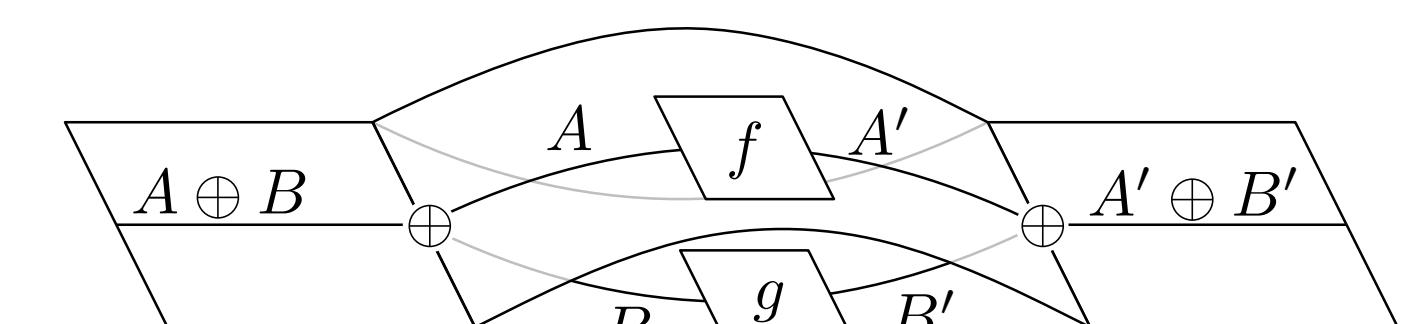
$$(\alpha|0\rangle + \beta|1\rangle) \otimes |y\rangle \mapsto \alpha|0\rangle \otimes (VU|y\rangle) + \beta|1\rangle \otimes (UV|y\rangle).$$

Not capturable by Quantum Circuits, QRAM Model

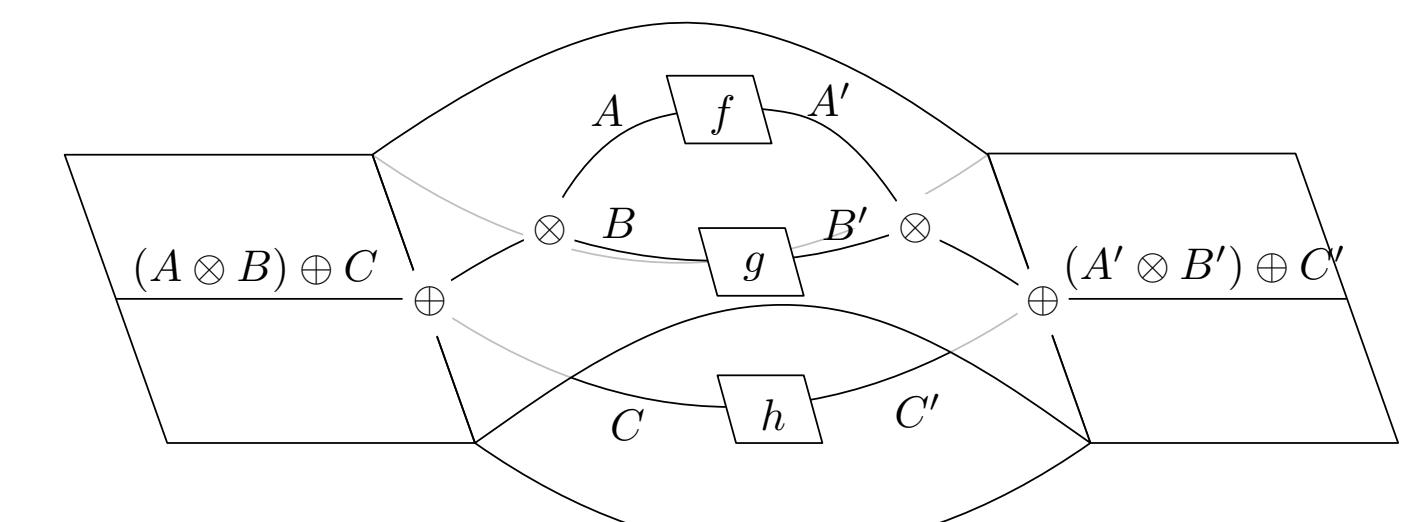
⇒ But physically realisable!
Need for better formalism

The Multi-World Calculus

- **Clear Semantics:** Denotational and Operational (Gol)
 - **Quantum Control:** Possible for every operator
 - **Quantum Types:** Not just qubits in superposition
- ⇒ Symmetric Monoidal Closed Category + Coproduct



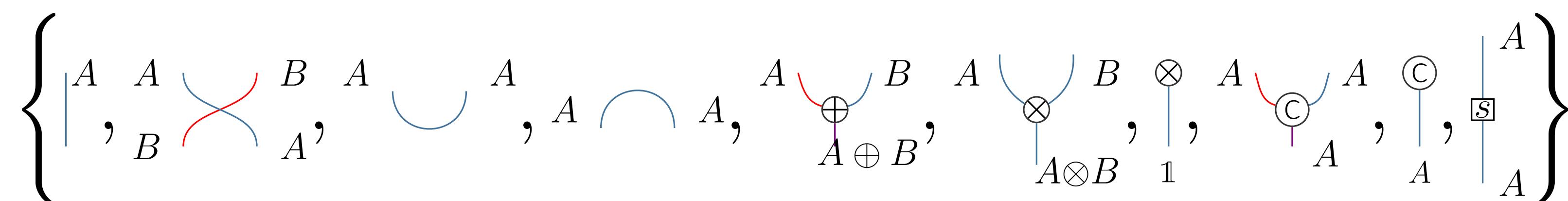
Split over coproduct



Splits over coproduct and tensor

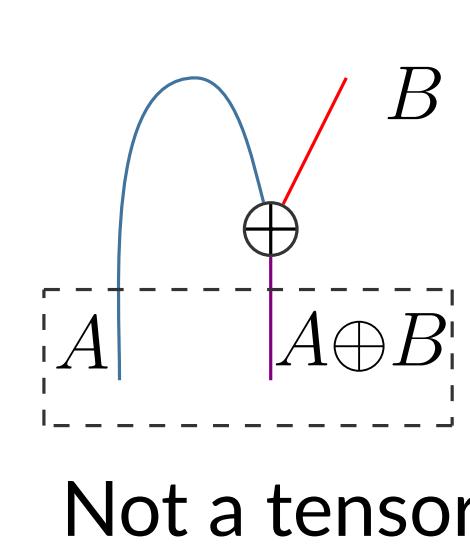
The Graphical Language

Types: $A, B ::= \mathbb{1} \mid A \oplus B \mid A \otimes B$



Composition:

$$D_2 \circ D_1 := \begin{array}{|c|c|} \hline \dots & D_1 \\ \hline \dots & D_2 \\ \hline \end{array} \quad D_1 \square D_2 := \begin{array}{|c|c|} \hline \dots & D_1 \\ \hline \dots & D_2 \\ \hline \end{array}$$



$A \square B :=$ either $(A \otimes B)$ or $(A \oplus B)$

Not a tensor

Semantics

- Denotational Semantics $\llbracket . \rrbracket : D \mapsto \mathcal{M}(\mathbb{C})$

- Universality: $\forall f : \mathbb{C}^n \mapsto \mathbb{C}^m, \exists D, \llbracket D \rrbracket = f$

$$\bullet \text{ World System: } \begin{array}{c} w \oplus v \\ \oplus \\ w \sqcup v \end{array} \quad \begin{array}{c} w \otimes w \\ \otimes \\ w \end{array}$$

- Equational Theory: ~ 15 axioms

$$\begin{array}{c} \text{Diagram showing equality between two graphical terms} \\ = \end{array} \quad \begin{array}{c} \text{Diagram showing equality between two graphical terms} \\ = \end{array}$$

- Soundness & Completeness:
 $\vdash D = D' \Leftrightarrow \llbracket D \rrbracket = \llbracket D' \rrbracket$

Examples

$$\alpha|0\rangle + \beta|1\rangle := \begin{array}{c} \otimes \\ \alpha \\ \oplus \\ \beta \end{array} \quad \left(\begin{array}{cc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{array} \right) := \begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}. \end{array}$$

$$\begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}. \\ = \end{array} \quad \begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}. \\ = \end{array} \quad \begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{\alpha}{\sqrt{2}}, \frac{\beta}{\sqrt{2}}, -\frac{\beta}{\sqrt{2}}, \frac{\alpha}{\sqrt{2}}. \\ = \end{array} \quad \begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{\alpha+\beta}{\sqrt{2}}, \frac{\alpha-\beta}{\sqrt{2}}, \end{array}$$

Back to the Quantum Switch

$$\begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}. \\ = \end{array} \quad \begin{array}{c} \text{Diagram showing the graphical representation of a 2x2 matrix with entries } \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}. \\ = \end{array}$$