Quantum metrology of indefinite causal order strategies

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Abstract

- Quantum mechanics allows different causal orders to be superposed, leading to a genuinely quantum lack of causal structure. For example, the process known as the quantum switch (QS) consists in the superposition of applying two operations A and B in their two possible orders, A after B and B after A.
- An advantage of such processes with indefinite causal order has been claimed in quantum metrology [1], solely on the grounds of a comparison between the QS and the sequential strategy. We first argue that such a claim does not hold.
- Using a framework introduced in [2,3], we then address the question of the comparison between processes with definite and indefinite causal order in quantum metrology.
- By introducing new sets of strategies, we extend a hierarchy found in [3]. We also show that the set of quantum circuits with quantum control of the causal order strictly outperforms any set with physically realizable strategies so far considered.

Quantum metrology

\[ \rho \xrightarrow{C_\theta} \rho \theta \]

FIG. 1: A quantum channel \( C_\theta \) that depends on an unknown parameter \( \theta \), with an input (resp. output) state \( \rho \) (resp. \( \rho_\theta \)). The objective is to gain some information about \( \theta \) by measuring the output state.

- The quantum Fisher information (QFI) of the output state \( \rho_\theta \) with respect to the unknown parameter \( \theta \) can be computed as:
  \[ J(\rho_\theta) = 4 \min_{(i_0,i_1)} \sum_i \text{Tr} \left( |\psi_{i_0}(\theta)\rangle \langle \psi_{i_1}(\theta)| \right), \]
  (1)
  where \( |\psi_{i_0}(\theta)\rangle \) is a set of unnormalized vectors such that \( \rho_\theta = \sum_i |\psi_{i_0}(\theta)\rangle \langle \psi_{i_1}(\theta)| \).
- The QS and the sequential strategy (Seq) were compared in [1], for \( N = 2 \) depolarizing channels: \( C_\theta(\rho) = (1-\theta) \text{Tr}(\rho_\theta^2) + \theta \rho \).

\[ \rho \xrightarrow{C_{\theta}} \rho \theta \xrightarrow{C_\theta} \rho \theta \]

FIG. 2: Three strategies for \( N = 2 \) copies of the quantum channel \( C_\theta \). (a) The QS strategy. The red (resp. blue) path corresponds to the evolution of the target system \( S \) when the control qubit \( C \) is in the state \( |0\rangle \) (resp. \( |1\rangle \)). (b) The sequential strategy. (c) A parallel strategy with initial entanglement (ParallelEnt), where \( |\psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \).

- On the grounds that \( J_{\text{Seq}}(\rho_\theta) > J_{\text{Ent}}(\rho_\theta), \forall \theta \in [0,1] \), [1] claimed that “indefinite causal order is an aid for channel probing”. Such a claim requires more general comparison between strategies with and without a definite causal order, since for instance we could show that \( J_{\text{Ent}}(\rho_\theta) > J_{\text{Seq}}(\rho_\theta), \forall \theta \in [0,1] \).
- What is the best strategy with (in)definite causal order?

A metrological task

Given \( N \) queries to a quantum channel \( C_\theta \) that depends on an unknown parameter \( \theta \), what is the strategy with (in)definite causal order that maximizes the QFI of the output state \( \rho_\theta \)?

\[ \rho \xrightarrow{C_{\theta}} \rho \theta \xrightarrow{C_{\theta}} \rho \theta \]

FIG. 3: Framework defining the metrological task for \( N = 1 \) queries to \( C_\theta \). Starting with an initial state \( \rho \), the strategy is connecting the \( N \) quantum channels \( C_\theta \) in a (in)definite causal order in order to output the state \( \rho_\theta \).

References