Quantum metrology of indefinite causal order strategies

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Abstract

- Quantum mechanics allows different causal orders to be superposed, leading to a genuinely quantum lack of causal structure. For example, the process known as the quantum switch (QS) consists in the superposition of applying two operations A and B in their two possible orders, A after B and B after A.
- An advantage of such processes with indefinite causal order has been claimed in quantum metrology [1], solely on the grounds of a comparison between the QS and the sequential strategy. We first argue that such a claim does not hold.
- Using a framework introduced in [2,3], we then address the question of the comparison between processes with definite and indefinite causal order in quantum metrology.
- By introducing new sets of strategies, we extend a hierarchy found in [3]. We also show that the set of quantum circuits with quantum control of the causal order strictly outperforms any set with physically realizable strategies so far considered.

Quantum metrology

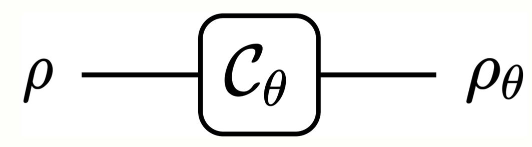

FIG. 1: A quantum channel $\mathcal{C}_\theta$ that depends on an unknown parameter θ, with an input (resp. output) state ρ (resp. ρ_θ). The objective is to gain some information about θ by measuring the output state.

- The quantum Fisher information (QFI) of the output state ρ_θ with respect to the unknown parameter θ can be computed as:

$$J(\rho_\theta) = 4 \min_{\{|\psi_{\theta,i}\rangle\}} \sum_i \mathrm{Tr}\left(\left|\dot{\psi}_{\theta,i}\right\rangle \left\langle \dot{\psi}_{\theta,i}\right|\right), \quad (1)$$

where $|\psi_{\theta,i}\rangle$ is a set of unnormalized vectors such that $\rho_\theta = \sum_i |\psi_{\theta,i}\rangle \langle\psi_{\theta,i}|$.
- The QS and the sequential strategy (Seq) were compared in [1], for $N = 2$ depolarizing channels: $\mathcal{C}_\theta(\rho) = (1-\theta)\,\mathrm{Tr}(\rho)\frac{\mathbb{1}}{2} + \theta\rho$.

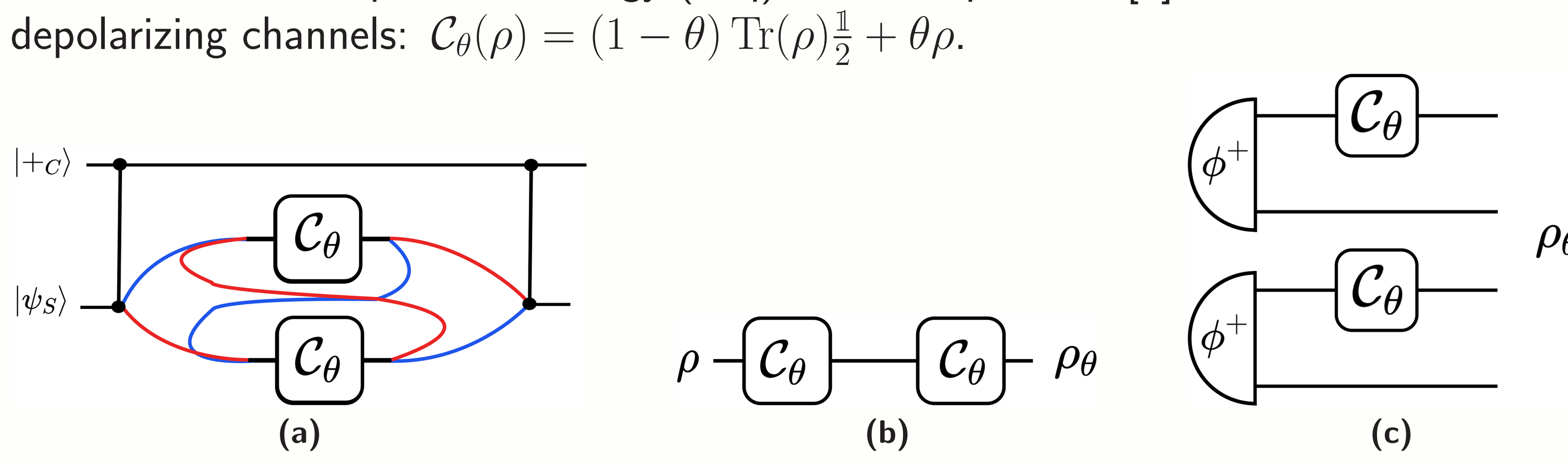

FIG. 2: Three strategies for $N = 2$ copies of the quantum channel $\mathcal{C}_\theta$. (a) The QS strategy. The red (resp. blue) path corresponds to the evolution of the target system S when the control qubit C is in the state $|0_C\rangle$ (resp. $|1_C\rangle$). (b) The sequential strategy. (c) A parallel strategy with initial entanglement (ParaEnt), where $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

- On the grounds that $J^{\text{QS}}(\rho_\theta) > J^{\text{Seq}}(\rho_\theta)$, $\forall\theta \in [0,1]$, [1] claimed that "indefinite causal order is an aid for channel probing". Such a claim requires a more general comparison between strategies with and without a definite causal order, since for instance we could show that $J^{\text{ParaEnt}}(\rho_\theta) > J^{\text{QS}}(\rho_\theta)$, $\forall\theta \in [0,1]$.
- What is the best strategy with (in)definite causal order?

A metrological task

Given N queries to a quantum channel $\mathcal{C}_\theta$ that depends on an unknown parameter θ, what is the strategy with (in)definite causal order that maximizes the QFI of the output state ρ_θ?

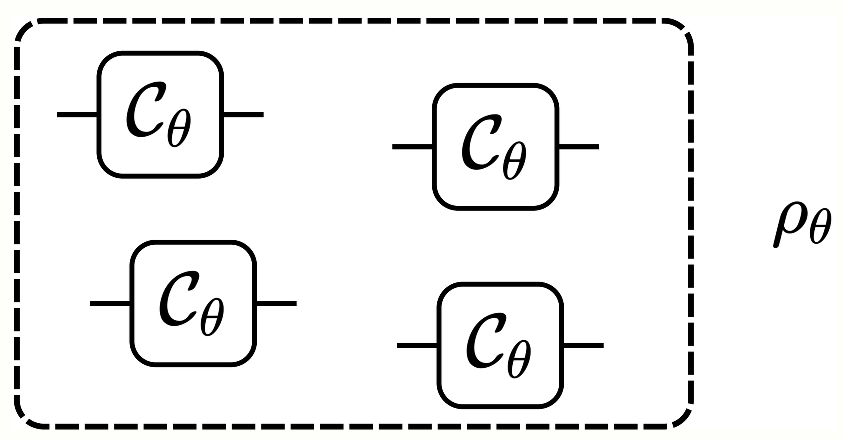

FIG. 3: Framework defining the metrological task for $N = 4$ queries to $\mathcal{C}_\theta$. Starting with an initial state ρ, the strategy is connecting the N quantum channels $\mathcal{C}_\theta$ in a (in)definite causal order in order to output the state ρ_θ.

References

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Description of processes with fixed causal order (FCO) and indefinite causal order (ICO)

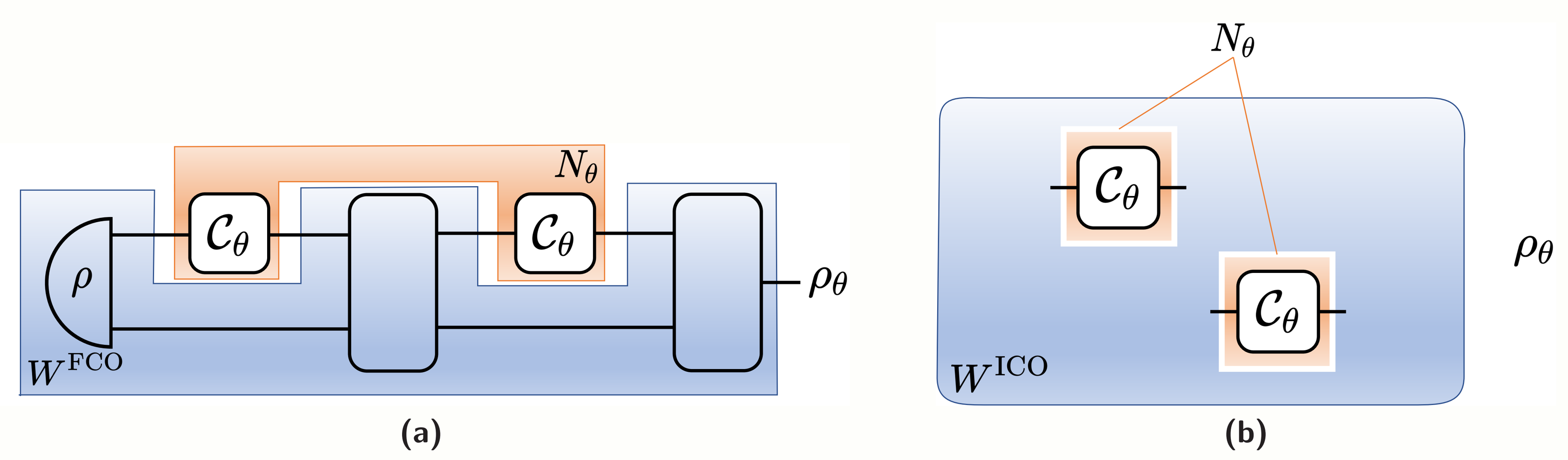

FIG. 4: The blue part corresponds to the process described by the process matrix W, while the orange part corresponds to the external channels $\mathcal{C}_\theta$ described by the operator N_θ, that are embedded in the process. (a) Process with fixed causal order. (b) Process with indefinite causal order.

- Rewriting the output state ρ_θ as the link product of W and N_θ, $\rho_\theta = W * N_\theta$, the optimal QFI over all FCO or ICO strategies may be computed as:

$$J^X(N_\theta) = \max_{W^X} J(W^X * N_\theta), \quad (2)$$

where $X = $ FCO, ICO.
- Eq. (2) can be computed via semidefinite programming methods [2,3].

Different sets of strategies

Three sets of strategies were compared in [3]:
- Quantum circuits with FCO (QC-FCO): fixed causal order.
- Quantum circuits with causal superposition (QC-CS): coherent superposition of different fixed causal orders.
- ICO strategies (ICO): all processes with indefinite causal order.

We consider two extra sets of strategies introduced in [4], that are physically realizable:
- Quantum circuits with classical control of the causal order (QC-CC): causal order not predetermined but not coherently superposed.
- Quantum circuits with quantum control of the causal order (QC-QC): causal order not predetermined and coherently superposed.

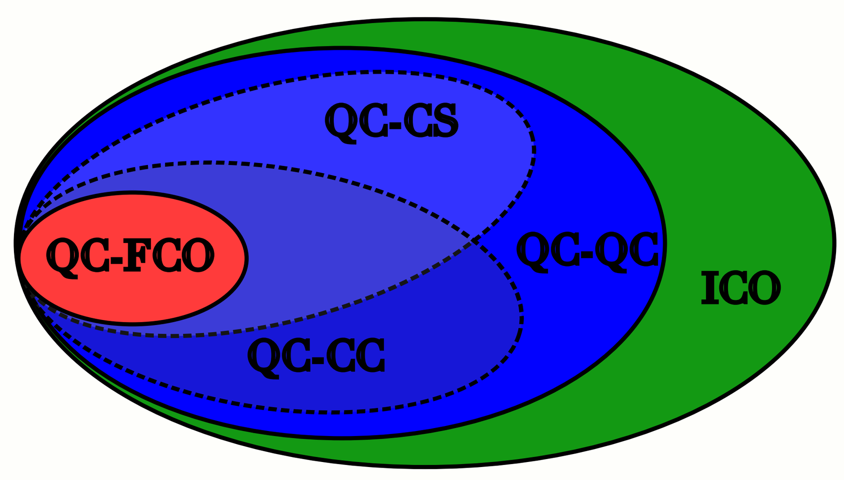

FIG. 5: Relation between the different strategies with or without definite causal order.

Comparison between the sets of strategies using the metrological task

- $N = 3$ amplitude damping channels, defined as a z-rotation of angle θ, $U_z(\theta) = e^{-i\theta\sigma_z/2}$, followed by a quantum channel described by the two Kraus operators $K_1 = |0\rangle\langle 0| + \sqrt{1-p}\,|1\rangle\langle 1|$ and $K_2 = \sqrt{p}\,|0\rangle\langle 1|$, with the decay parameter p.

$$J^{\text{QC-FO}}(N_\theta) < J^{\text{QC-CC}}(N_\theta) = J^{\text{QC-CS}}(N_\theta) < J^{\text{QC-QC}}(N_\theta) < J^{\text{QC-ICO}}(N_\theta), \quad (3)$$

$\forall p \in [0,1]$.
- $N = 2$ depolarizing channels

$$J^{\text{QC-FO}}(N_\theta) = J^{\text{QC-CC}}(N_\theta) = J^{\text{QC-CS}}(N_\theta) = J^{\text{QC-QC}}(N_\theta) = J^{\text{QC-ICO}}(N_\theta). \quad (4)$$

$\rightarrow$ Contrary to claim of [1], no advantage from ICO strategies.
- $N = 3$ depolarizing channels

$$J^{\text{QC-FO}}(N_\theta) < J^{\text{QC-CC}}(N_\theta) = J^{\text{QC-CS}}(N_\theta) = J^{\text{QC-QC}}(N_\theta) = J^{\text{QC-ICO}}(N_\theta). \quad (5)$$

Conclusion

- Framework to compare different sets of strategies with (in)definite causal order on a metrological task.
- Strict advantage of QC-QCs among physically realizable strategies so far considered.
- Relation between QC-CCs and QC-CSs?
- No advantage of ICO strategies for N depolarizing channels?

This work is supported by the Agence Nationale de la Recherche under the programme "Investissements d'avenir" (ANR-15-IDEX-02).

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