Quantum Phase Estimation in the Presence of Noise

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Abstract

The quantum phase estimation (QPE) is one of the fundamental algorithms based on the quantum Fourier transform (QFT). It has applications in order-finding, factoring, and finding the eigenvalues of unitary operators. The major challenge in running QPE and other quantum algorithms is the noise in quantum computers. This noise is due to the interactions of qubits with the environment and due to the faulty gate operations. In the present work, we study the impact of incoherent noise on QPE, modeled as trace-preserving and completely positive quantum channels. Different noise models such as depolarizing, phase flip, bit flip, and bit-phase flip are taken to understand the performance of the QPE in the presence of noise. The simulation results indicate that the standard deviation of the eigenvalue of the unitary operator has strong exponential dependence upon the error probability of individual qubits. Furthermore, the standard deviation increases with the number of qubits for fixed error probability.

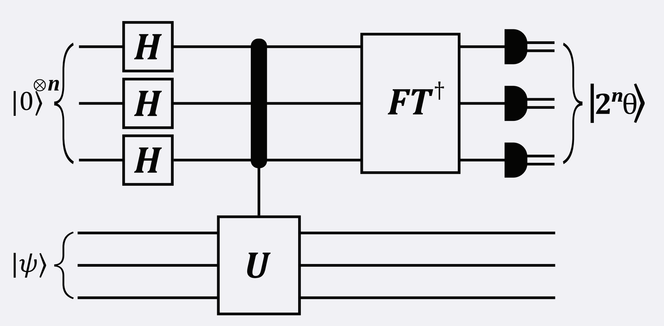
Modeling noise in QPE

The QPE algorithm finds the eigenvalue of a unitary operator U such that

$$U|v\rangle = e^{2\pi i\theta}|v\rangle \tag{1}$$

where $0 \le \theta \le 1$. The goal of this algorithm is to provide an n-bit approximation of θ in a single run. This algorithm uses two registers. The first register is initialized to $|0\rangle^{\otimes n}$ and is called the register of controlled qubits, and the second register is initialized to $|v\rangle$, and it contains the number of qubits necessary to store $|v\rangle$. The algorithm is performed in four steps:

1. Putting the qubits of first register into a uniform superposition of all the computational basis states.
2. Application of controlled-Unitary operation U^{2^j}.
3. Applying $FT^\dagger$ to the first register and measuring it.
4. Classical post-processing to extract estimated θ.



In the presence of noise, qubit's evolution is no longer unitary. Instead, its evolution can be captured using completely positive and trace-preserving maps of the density operators, commonly known as quantum channels. For the simulation results in this paper, we considered four noisy quantum channels to represent noise processes.

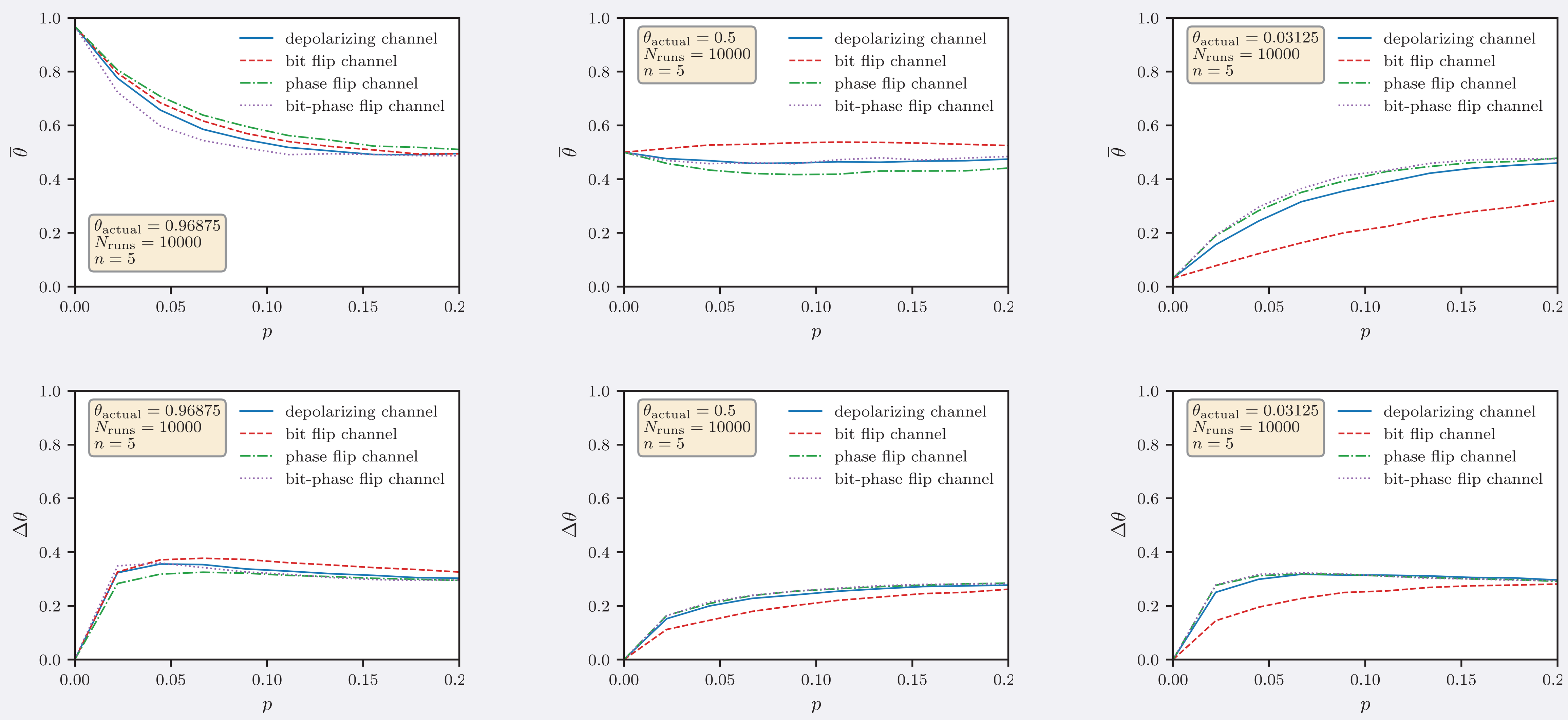
1. Depolarizing channel
2. Bit flip channel
3. Phase flip channel
4. Bit-phase flip channel

To investigate the performance of QPE in the presence of noise, we transpiled the circuit to the one containing basic gates set $\{I, X, \sqrt{X}, R_z, CX\}$ and then incorporated one of the noise models into the circuit that adds errors to all the basic gates acting on each qubit.

Simulation Results and Discussions

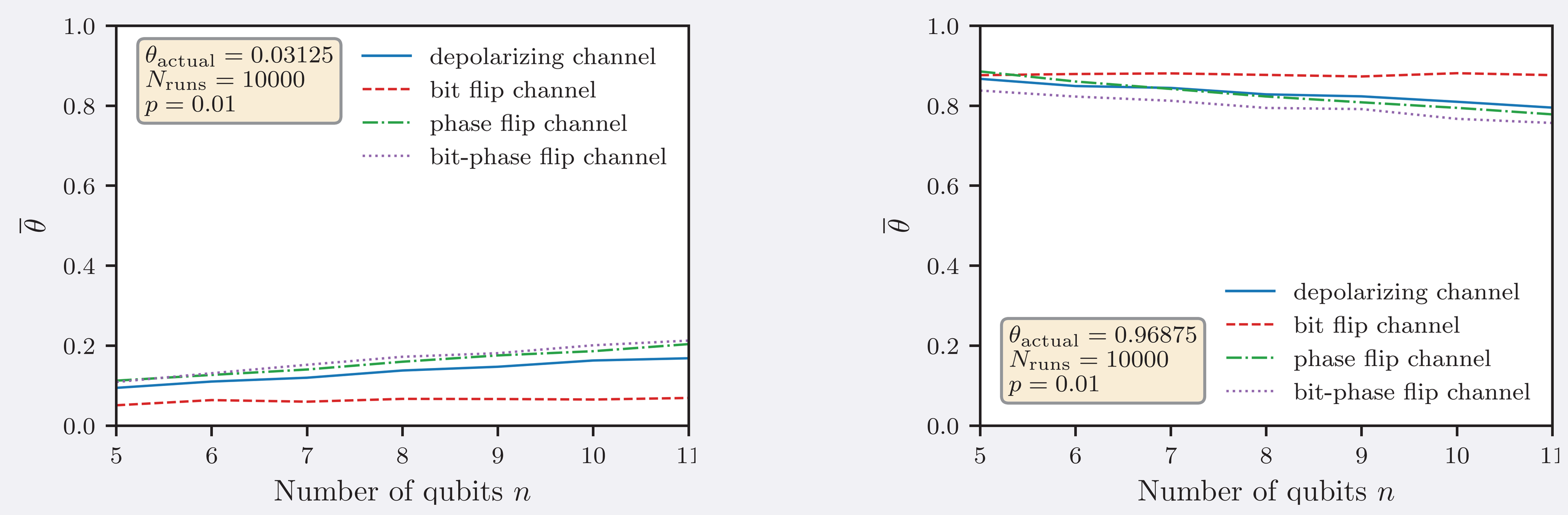
- We ran the QPE with noise for three different actual values θ_{actual} vs. error probability p.
- We computed the average value of θ as $\overline{\theta}$, and standard deviation as $\Delta\theta$ by running the algorithm N_{runs} number of times for each value of p.
- Results indicate that the average value of θ approaches 0.5 as the error probability increases.
- Also, the standard deviation of the results increases and then saturates. This is consistent with the interpretation of the noise processes leading toward depolarization of the quantum state in the circuit.

Impact of Noise on QPE for fixed number of qubits



Average value $\overline{\theta}$ and standard deviation $\Delta\theta$ plotted as function of error probability p for three different values of the actual θ when QPE is implemented with $n = 5$ qubits.

Impact of Number of Qubits on QPE for fixed error probability



Average value $\overline{\theta}$ as function of number of qubits n for fixed error probability: p and for two distinct values of actual θ.

Impact of Noise on QPE for small range of error probability

To model the dependence of standard deviation $\Delta\theta$ as a function of p, when p is small, we fitted the data to the function

$$\Delta\theta(p) = k_1 + k_2 e^{-k_3 p}, \quad 0 \le p \le 0.01, \tag{2}$$

for all four channels and obtained parameters of the curve that best models the data with more than 97.5% accuracy. The results showed that $\Delta\theta$ has strong exponential dependence upon p.

Conclusions

The simulation results indicated that the average value of the eigenvalue of the unitary operator converged to 0.5 regardless of the actual value of the eigenvalue as the error probability increases from 0, indicating that the noise processes force the overall quantum state towards the maximally mixed state. The standard deviation of the output increased exponentially for a small value of p as p increased from 0. Furthermore, the average value of the eigenvalue diverged away from the actual value when the number of qubits was increased for fixed error probability, contrary to the noiseless QPE where increasing the number of qubits increases the precision of the eigenvalue.