Quantum network for detecting and activating entanglement

Jaemin Kim† and Joonwoo Bae

The School of Electrical Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Republic of (South) Korea

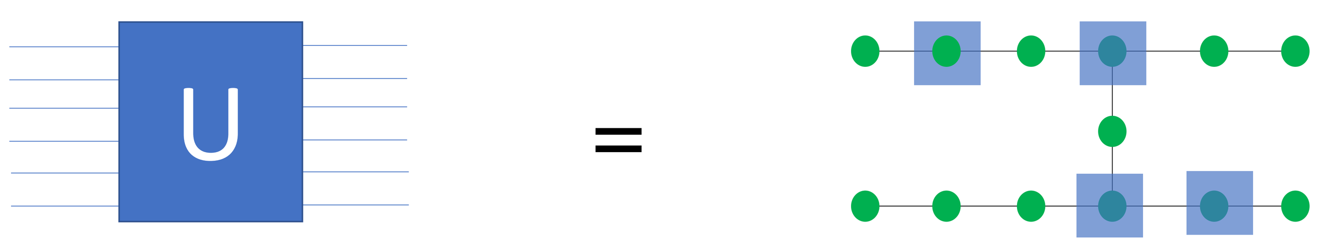
Abstract : We present measurement-based entanglement witnesses that realize the detection of entangled states by preparing multipartite quantum states and applying local measurements only, where a measurement setup for the verification of entanglement is hugely simplified. Entanglement witnesses from decomposable maps such as the partial transpose and the reduction map, which detect distillable entangled states, and non-decomposable positive maps such as the Choi map as well as its generalizations and the Breuer-Hall map, which verify undistillable entangled states, are explicitly constructed in a measurement-based manner. The realization is also applied to characterizing entangled states that can be used for activation of quantum information processing. Our results generalize measurement-based quantum computing to non-physical operations that can detect entangled states, and establish both detection and activation of entangled states in a measurement-based manner.

† kimjm.kaist@gmail.com

Introduction

Motivation

- Measurement-Based Quantum State Transformation (State-Channel Duality)

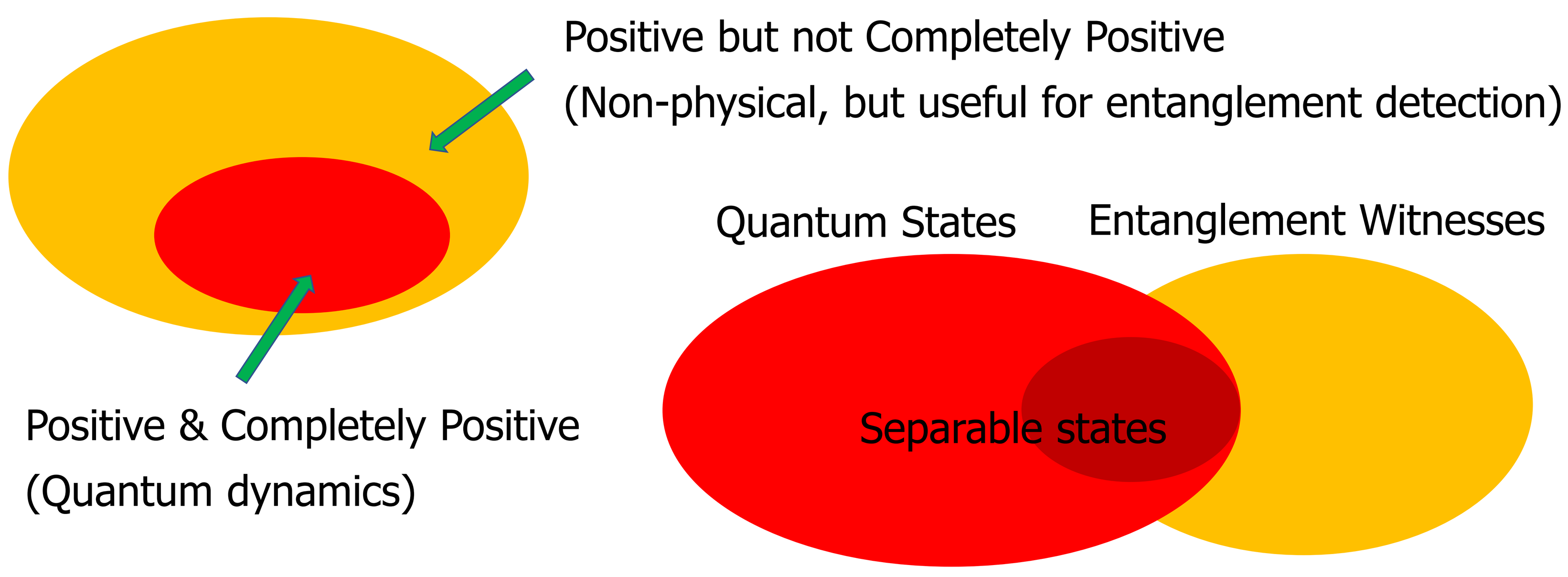


Quantum channel = Entanglement States and Local Measurements

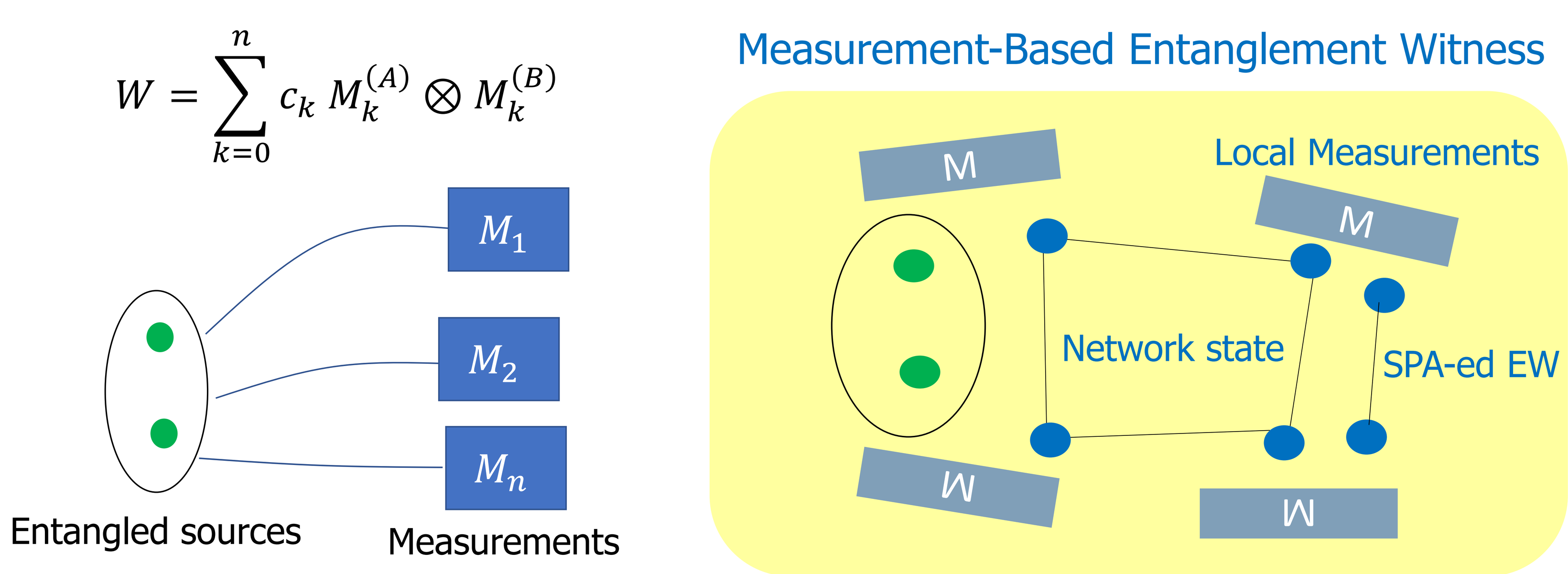
- The Mathematical Result

Choi–Jamiołkowski Isomorphism		Implementation with State + Measurement
Map	Choi Matrix	
P & CP	Quantum State	MBQC
P & not CP	EW	?

Positive Maps and Linear Operators

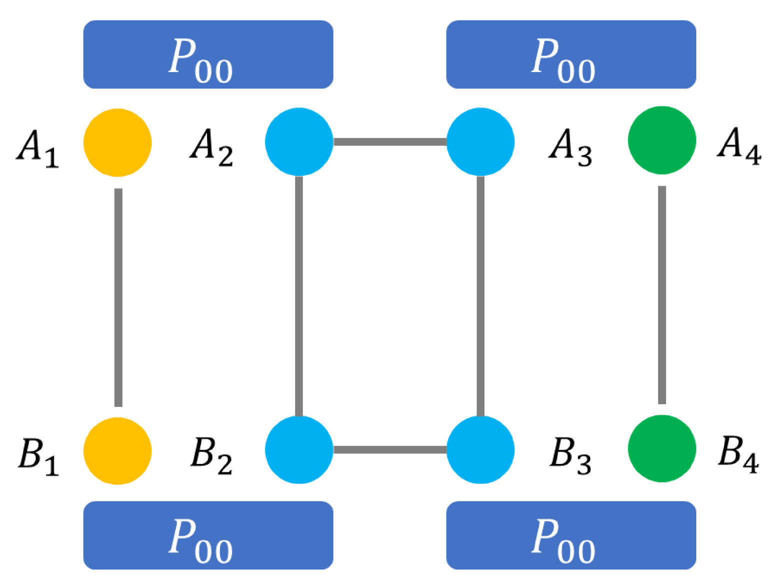


Entanglement Witnesses in Experiment



The framework

- Projective measurements $P_{00} = |\phi^+\rangle\langle\phi^+|$ are performed at $A_1A_2, A_3A_4, B_1B_2, B_3B_4$
- $\widetilde{W}_\mu^{(A_1B_1)} \otimes \rho^{(A_2B_2A_3B_3)} \otimes \sigma^{(A_4B_4)}$: A network for entanglement detection & activation
  - $\widetilde{W}_\mu^{(1)}$: the SPA-ed EW to test the singlet fraction of $\rho^{(23)} \otimes \sigma^{(4)}$
  - $\rho^{(23)}$: A state to be activated
  - $\sigma^{(4)}$: A state to be detected / A state consumed for activation
- → direction: entanglement detection of $\sigma^{(4)}$
- ← direction: entanglement activation of $\rho^{(23)}$



Results

Definitions and Notations

- d-dimensional Bell states
  - $|\phi_{00}\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle|k\rangle$, $|\phi_{st}\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \omega^{kt} |k\rangle|k \oplus s\rangle$, $\mathrm{P}_{st} = |\phi_{st}\rangle\langle\phi_{st}|$
- Separable Bell-diagonal projectors
  - $\Pi_s = \sum_{t=0}^{d-1} P_{st} = \sum_{k=0}^{d-1} |k\rangle\langle k| \otimes |k \oplus s\rangle\langle k \oplus s|$
- The maximal singlet fraction (MSF) of a state ρ
  - $MSF(\rho) = \sup_{\Omega \in \mathrm{SEP}} \frac{\mathrm{tr}[\Omega(\rho)\, P_{00}]}{\mathrm{tr}[\Omega(\rho)]}$, $MSF(\rho) \in \left[\frac{1}{d}, 1\right]\ \forall \rho$
  - The largest overlap with P_{00} achievable by separable map (SEP)
- The direct singlet fraction (DSF) of a state ρ: a lower bound of $MSF(\rho)$
  - $DSF(\rho) = \frac{\mathrm{tr}[\Lambda(\rho)\, P_{00}]}{\mathrm{tr}[\Lambda(\rho)]}$ for some trivial separable map Λ. $DSF(\rho) \le MSF(\rho)$.

Main Results

- Any entangled state σ can be detected by some EW W' which is constructed as
  - $W' = \mathrm{tr}_2\left[W_\eta^{(2)} \rho^{\mathrm{T}(23)}\right]$ where $W_\eta = \eta \mathbb{1} - P_{00}$, $\eta = MSF(\rho)$ [1]
- If an EW W is given, we can utilize DSF instead of MSF.
- Construct a 4–partite state $\rho^{(A_2B_2A_3B_3)}$ such that
  - $W = \mathrm{tr}_2\left[W_\mu^{(2)} \rho^{\mathrm{T}(23)}\right]$ where $W_\mu = \mu\mathbb{1} - P_{00}$, $\mu = DSF(\rho)$
  - $\mathrm{tr}[W\sigma] = d^4 \mathrm{tr}\left[W_\mu^{(1)} \otimes \rho^{(23)} \otimes \sigma^{(4)} P_{00}^{\otimes 4}\right]$ where $\rho = \rho^{\mathrm{T}}$ (real coefficients)
- **Theorem.** If W is an EW, then $\mathrm{tr}[W\sigma] < 0 \Leftrightarrow DSF(\rho \otimes \sigma) > \mu$
  - σ is detected by W if and only if σ activates ρ
- **Remark.** If ρ_{PPT} is a PPT state, then the followings hold
  - $MSF(\rho_{PPT}) = 1/d$ since ρ_{PPT} is undistillable.
  - $W_{PPT} = \mathrm{tr}_2\left[W_{1/d}^{(2)} \rho_{PPT}^{(23)}\right]$ is decomposable (cannot detect PPTES).
  - Note. Any PPT entangled state $\sigma^{(4)}$ cannot activate a PPT state $\rho_{PPT}^{(23)}$.
- **Transposition EW** $W_T = (\mathrm{id} \otimes T)(d\, P_{00}) = \mathbb{F} = \sum_{i,j=0}^{d-1} |i\rangle\langle j| \otimes |j\rangle\langle i|$
  - $\rho_T = \frac{2}{(d-1)d(d+1)(d+2)}\left[(d+1)P_{00}^{(2)} \otimes \left(\frac{\mathbb{1}-\mathbb{F}}{2}\right)^{(3)} + (\mathbb{1} - P_{00})^{(2)} \otimes \left(\frac{\mathbb{1}+\mathbb{F}}{2}\right)^{(3)}\right]$
  - $MSF(\rho_T) = 1/d$ since ρ_T is undistillable. [2]
  - $DSF\left(\rho_T^{(23)} \otimes \sigma^{(4)}\right) > \frac{1}{d} \Leftrightarrow \mathrm{tr}[W_T\sigma] < 0$
- **Reduction EW** $W_R = \frac{1}{d}\mathbb{1} - P_{00} = \sum_{s=0}^{d-1} \frac{1}{d} \Pi_s - P_{00}$
  - $\rho_R = \frac{1}{d^2} \sum_{s=0}^{d-1} \sum_{k=0}^{d-1} P_{sk}^{(2)} \otimes P_{sk}^{(3)}$ (d-dimensional Smolin state), $DSF(\rho_R) = 1/d$
- **Generalized d-dimensional Choi EW** $W_{GC} = \frac{\alpha_0+1}{d}\Pi_0 + \sum_{s=1}^{d-1} \frac{\alpha_s}{d} \Pi_s - P_{00}$
  - $\rho_{GC} = \frac{\alpha_0+1}{d^2} \sum_{k=0}^{d-1} P_{0k}^{(2)} \otimes P_{0k}^{(3)} + \sum_{s=1}^{d-1} \frac{\alpha_s}{d^2} \sum_{k=0}^{d-1} P_{sk}^{(2)} \otimes P_{sk}^{(3)}$
  - $DSF(\rho_{GC}) = \frac{\alpha_0+1}{d}$, $\Lambda(\rho_{GC}) = \mathrm{tr}_{34}\left[\rho_{GC}^{(23)} \otimes \left(\frac{1}{d}P_{00} + \frac{1}{d}\sum_{s=1}^{d-1} \frac{\Pi_s}{d}\right)^{(4)} \left(P_{00}^{\otimes 2}\right)^{(34)}\right]$ [3]
  - $DSF\left(\rho_{GC}^{(23)} \otimes \sigma^{(4)}\right) > \frac{\alpha_0+1}{d} \Leftrightarrow \mathrm{tr}[W_{GC}\sigma] < 0$
- **Breuer-Hall EW** $W_{BH} = \frac{1}{d}\mathbb{1} - P_{00} - \frac{1}{d}\mathbb{F}'$
  - $\mathbb{F}' = (\mathbb{1} \otimes U)\mathbb{F}(\mathbb{1} \otimes U^\dagger)$, U is any skew-symmetric unitary: $UU^\dagger = \mathbb{1}, U^{\mathrm{T}} = -U$
  - $\rho_{BH} = t \cdot \frac{1}{d^2} \sum_{s=0}^{d-1} \sum_{k=0}^{d-1} P_{sk}^{(2)} \otimes P_{sk}^{(3)}$ $+ (1-t) \cdot \frac{2}{(d-1)d(d+1)(d+2)}\left((d+1)P_{00}^{(2)} \otimes \left(\frac{\mathbb{1}+\mathbb{F}'}{2}\right)^{(3)} + (\mathbb{1} - P_{00})^{(2)} \otimes \left(\frac{\mathbb{1}+\mathbb{F}'}{2}\right)^{(3)}\right)$
  - $t = \frac{2d^2-2d}{3d^2-3d+2}$, $DSF(\rho_{BH}) = \frac{1}{d}$
  - $DSF\left(\rho_{BH}^{(23)} \otimes \sigma^{(4)}\right) > \frac{1}{d} \Leftrightarrow \mathrm{tr}[W_{BH}\sigma] < 0$

References & Acknowledgement

[1] L. Masanes, "All bipartite entangled states are useful for information processing." *Physical Review Letters* 96.15 (2006).
[2] K. Vollbrecht and M. Wolf, Vollbrecht, "Activating distillation with an infinitesimal amount of bound entanglement." *Physical review letters* 88.24 (2002).
[3] D. Chruściński and A. Kossakowski, "Bell diagonal states with maximal abelian symmetry." *Physical Review A* 82.6 (2010)

This work is supported by the National Research Foundation of Korea (NRF-2021R1A2C2006309).

Conclusion

- We provide the quantum network for entanglement detection and activation.
- Well-known EWs from transposition map, reduction map, Bell-diagonal map, and Breuer-Hall map are shown in measurement-based entanglement detection.
- Highly noisy network states construct non-trivial EWs.
- Future works: The properties of EWs (e.g., optimality, atomicity) in the quantum network